NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3140

USE OF AERODYNAMIC HEATING TO PROVIDE THRUST BY VAPORIZATION OF SURFACE COOLANTS

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SUMMARY

Analysis is made of the propulsive thrust and specific impulse attainable by cooling aircraft surfaces with liquefied or solidified gases having vaporization temperatures lower than the equilibrium surface temperature. It is found that for some coolants a net thrust is obtainable without further heat addition at all flight speeds if the vaporization temperature and heat of vaporization are sufficiently low.

Use of coolant vaporization as the sole means of propulsion yields low specific impulses at low speeds. As flight speed increases, the attainable specific impulse increases; if liquid hydrogen is used as coolant, specific impulses comparable to those of conventional liquid rocket propellants are theoretically attainable in the hypersonic speed range. This propulsion system provides the possibility of flight with cooled aircraft surfaces at extremely high Mach numbers.

As an auxiliary power source, hydrogen vaporization by aerodynamic heating can maintain surface temperatures sufficiently low to preserve structural integrity at all flight speeds, and the gas can be ejected with a specific impulse in the range of current liquid rocket propellants.

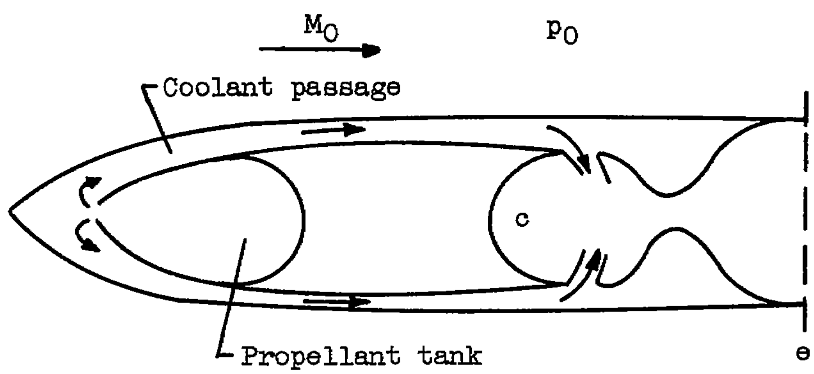
INTRODUCTION

Rocket propulsion is based on the conversion of liquid or solid fuel into gases which are expelled rearward at the highest possible speed. Normally, chemical energy of the fuel is utilized to produce the desired gas phase, and to raise the gas temperature to produce high jet velocities.

In the present report it is proposed that aerodynamic (or atmospheric) heating be used to vaporize and heat a liquefied or solidified gas having a vaporization temperature below the equilibrium temperature of the aircraft surfaces. The resulting gas is to be collected in a chamber and expelled through a nozzle as in conventional rocket propulsion. The thrust and specific impulse attainable with such a propulsion system are determined as function of Mach number for a variety of possible coolants. Use of coolant vaporization as an independent or as an auxiliary power plant is discussed, and the design parameters that determine the efficiency of the system are analyzed. Many engineering aspects of the system, however, are not considered herein. 2888

ANALYSIS

To arrive at an estimate of the thrust obtainable by the coolant vaporization technique, consider a missile such as that shown in the following sketch. A liquefied or solidified gas having a boiling point or



sublimation point considerably below equilibrium surface temperature is circulated or stored between the outer and the inner shell. The vaporized gases are collected in a chamber C and are allowed to expand through the nozzle. The liquid or solid phase is assumed to be at the equilibrium vaporization temperature corresponding to the chamber pressure p_c, and the gas phase is assumed to be raised to a final mean temperature equal to the chamber temperature T_c.

The total heat entering the coolant through the external surfaces is $q_w A_w$ Btu per second. (Symbols are defined in appendix A.) This heat vaporizes w pounds per second of the liquefied or solidified gas and raises the gas temperature from the vaporization temperature T_v to a final temperature T_c. The heat balance is therefore:

$$q_w A_w = w\left[h_v + c_{p,c}\,(T_c - T_v)\right] \tag{1}$$

where h_v is heat of vaporization. The mean rate of heat flow into the coolant, per square foot of external surface area, can be written

$$q_w = h_w(T_e - T_w) = \rho_0\,V_0\,c_{p,0}\,St_0\,(T_e - T_w) \tag{2}$$

where $T_e = T_0\left(1 + \frac{\gamma - 1}{2}\,\varepsilon\,M_0^{\,2}\right)$

In this equation, St is the Stanton number, T_e is the equilibrium surface temperature for an insulated surface, ε is the recovery factor, and T_w is the mean temperature of the entire cooled surface.

In terms of the specific impulse I of the gases at chamber conditions and the total weight flow w, the thrust is

$$\text{Thrust} = Iw \tag{3}$$

The net axial force on the missile, in horizontal flight, is

$$F_{net} = Iw - \text{Drag} = Iw - \left(C_{D,1} + \frac{A_w}{A_0}\,C_f\right)\frac{\rho_0 V_0^{\,2} A_0}{2g} \tag{4}$$

where $C_{D,1}$ is the total drag coefficient of the missile (based on A_0) with the exception of the friction drag of the cooled surfaces, C_f is the mean skin-friction coefficient for the cooled surfaces, and A_0 is the maximum frontal area. Use of equations (1) and (2) results in the following form of equation (4):

$$\frac{F_{net}}{\frac{1}{2}\,\frac{\rho_0}{g}\,V_0^{\,2}A_0} \equiv C_{F,net} = C_f\left[\frac{A_w}{A_0}\,(G - 1) - \frac{C_{D,1}}{C_f}\right] \tag{5}$$

where G is the ratio of thrust generated per unit area of the cooled surface to the friction drag of that unit area, and is expressed as:

$$G = \frac{Iw/A_w}{\frac{1}{2}\,\frac{\rho_0}{g}\,V_0^{\,2}C_f} = \left(\frac{gI}{V_0}\right)\left(\frac{St_0}{\frac{1}{2}\,C_f}\right)\frac{c_{p,0}(T_e - T_w)}{h_v + c_{p,c}(T_c - T_v)} \tag{6}$$

For turbulent boundary layers, the ratio $St/\frac{1}{2}\,C_f$ is found in reference 1 to be about 1.2; for laminar boundary layers (ref. 2) this ratio, for Prandtl number of 0.72, is about 1.245. Inasmuch as these values are almost the same, the value 1.2 will be used for both turbulent and laminar flow. Similarly, the recovery factor, which is about 0.845 and 0.88 for laminar and turbulent flow, respectively, will be assumed to be 0.88 for both types of boundary layer. With these assumptions, G is identical for laminar and turbulent flow.

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Equations (5) and (6) indicate that, for $T_w < T_e$, it should be possible to reduce the net drag of a missile by utilizing aerodynamic heating to vaporize a coolant. In particular, if G can be made equal to unity, the friction drag of the cooled surface is exactly cancelled. If G can be made sufficiently greater than unity, it should be possible to make the bracket in equation (5) equal to or greater than zero, in which case a net thrust can theoretically be attained. It is shown in appendix B that values of G greater than unity are compatible with the first law of thermodynamics.

Evaluation of specific impulse I and thrust parameter G. - The quantity gI, which is the effective jet velocity, can be written

$$gI = \varphi \sqrt{\frac{RT_c}{m_c}} \tag{7}$$

where m_c is the molecular weight of the propellant gas and φ is a dimensionless function of exit pressure p_e, chamber pressure p_c, ambient pressure p_0, and the ratio of specific heats of the propellant γ_c:

$$\varphi = \varphi_{max}\left[\frac{1 - \dfrac{\gamma_c + 1}{2\gamma_c}\left(\dfrac{p_e}{p_c}\right)^{\frac{\gamma_c-1}{\gamma_c}}\left(1 + \dfrac{\gamma_c - 1}{\gamma_c + 1}\dfrac{p_0}{p_e}\right)}{\sqrt{1 - \left(\dfrac{p_e}{p_c}\right)^{\frac{\gamma_c-1}{\gamma_c}}}}\right] \tag{8}$$

where $\varphi_{max} = \sqrt{\dfrac{2\gamma_c}{\gamma_c-1}}$.

For expansion to ambient pressure ($p_e = p_0$), equation (8) reduces to

$$\varphi = \varphi_{max}\sqrt{1 - \left(\frac{p_e}{p_c}\right)^{\frac{\gamma_c - 1}{\gamma_c}}} \tag{8a}$$

The maximum value of φ (and consequently the ideal value of I) is obtained when $p_e/p_c = 0$. Values close to this ideal may be attainable at extremely high altitudes. This ideal specific impulse is plotted in figure 1 as a function of final gas temperature T_c, for several possible coolants.

For hydrogen and helium, the specific impulse obtained when $p_e/p_c = 1/6$ is also plotted to indicate values corresponding to pressure ratios that should be readily attainable at lower altitude.

Of the coolants considered, only hydrogen and helium yield specific impulses in the range of current liquid propellants. (Although the liquid hydrogen - liquid oxygen propellant systems and a few others can attain specific impulses in the range 300 to 400 lb-sec/lb, most current systems are in the range 200 to 300 lb-sec/lb.) The conclusion to be drawn from figure 1 is that the coolant-vaporization method of propulsion can have propellant consumption comparable to that of conventional liquid propellants if hydrogen or helium is used and if chamber temperatures considerably higher than the vaporization temperatures are attained.

From equation (6), the following dimensionless thrust parameter can be derived:

$$\frac{GM_0}{\left(\frac{T_e - T_w}{T_0}\right)} = 1.2\left(\frac{gI}{a_0}\right)\left(\frac{c_{p,0}T_0}{h_v + c_{p,c}(T_c - T_v)}\right) \tag{9}$$

where T_0 and a_0 are ambient static temperature and sound speed, respectively. For a given ambient temperature, this parameter depends only on the propellant characteristics and on the ratios p_e/p_c and p_0/p_e. In figure 2, this parameter is plotted against chamber temperature for the propellants used in figure 1. The ambient temperature of the isothermal region of the atmosphere (35,000 to 100,000 ft) has been assumed. The data used to plot these curves (as well as those of fig. 1) are given in the following table, and were obtained from reference 3. These values of h_v and T_c correspond to pressures close to atmospheric; γ_c and $c_{p,c}$ are mean values appropriate to the low temperature range.

Substance	h_v, Btu/lb	$c_{p,c}$, Btu/lb-°R	T_v, °R	m_c	γ_c
A	67.7	0.133	151	40	1.67
CO	90.8	.259	149	28	1.41
CO_2	160	.184	350	44	1.3
He	10.8	1.25	7.2	4	1.67
H_2	194.5	2.64	36	2	1.5
N_2	85.7	.256	139	28	1.4
O_2	91.7	.228	162	32	1.4
NH_3	588	.523	432	17	1.31
H_2O	970	.482	672	18	1.31

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Figure 2 shows that helium, because of its low heat of vaporization, yields by far the largest thrust parameter at temperatures near the vaporization temperature. At higher temperatures, however, where the specific impulse is higher, helium and hydrogen yield considerably lower values of the thrust parameter than other substances. These low values are due primarily to the large specific heats.

From figure 2, the ratio of thrust to friction drag developed by a cooled surface (G) can be computed as a function of Mach number when the mean surface temperature and chamber temperature are specified. In figure 3(a), results are shown for a chamber temperature and mean surface temperature equal to the ambient temperature. For this case, argon and nitrogen provide the possibility of obtaining a net thrust $(G>1)$ for Mach numbers greater than about 3.0, while helium and hydrogen produce ideal values of G greater than unity for Mach numbers greater than 4.8 and 6.6, respectively. For the lower pressure ratio $(p_e/p_c = 1/6)$, the Mach numbers for which G equals unity for helium and hydrogen are 6.8 and 9.7, respectively.

Since the thrust parameter of figure 2 either changes little with T_c or reaches a maximum when $T_c = T_v$, the maximum values of G are approximately attained when the chamber temperature and the mean surface temperature are as low as possible, that is, equal to T_v. These values, however, are not attainable experimentally, since expansion through the nozzle would produce recondensation of the gas. The maximum values of G are nevertheless shown in figure 3(b) as a limiting case, and to show the variation of G with Mach number when the mean surface temperature T_w (assumed equal to T_c) is less than ambient temperature T_0. If T_e is expressed in terms of Mach number in equation (9), it is evident that G becomes infinite for $M_0 \rightarrow 0$ as well as for $M_0 \rightarrow \infty$ if $T_w < T_0$.

For the limiting case shown in figure 3(b), as could be anticipated from figure 2, helium produces by far the greatest thrust. Hydrogen, argon, and nitrogen, however, all yield maximum values of G greater than unity for all Mach numbers if the maximum value of φ can be approximately attained.

Shown in figure 3(c) are values of G obtained when the mean surface temperature and the chamber temperature are permitted to rise
2888 to 1200° R. Although the specific impulse for this case is much higher than for low surface temperatures, the value of G is much smaller at each Mach number.

The conclusion to be drawn from figure 3 is that aerodynamic heating can vaporize several liquefied gases at a rate sufficient to produce a net thrust at all Mach numbers if sufficiently low values of T_w and T_c are attainable. Such low values of T_c, however, produce low specific impulse. It is therefore evident that use of coolant vaporization as an independent propulsion system can be competitive with conventional rocket propulsion only at very high Mach numbers, where values of $G>1$ are attainable with higher values of T_c.

Hydrogen or helium vaporization can be used as an auxiliary propulsion system (for which large values of G are not required) with specific impulses comparable with those attained with conventional liquid propellants at all Mach numbers, since the magnitude of T_w (and consequently T_c) is limited only by the requirement that structural integrity be maintained.

Values of G and A_w/A_0 required for propulsion. - Although values of G greater than unity were found to be obtainable by vaporization of surface coolants at all Mach numbers, equation (5) shows that the condition

$$\frac{A_w}{A_0}(G-1) \geq C_{D,1}/C_f \tag{10}$$

must be satisfied if propulsion by coolant vaporization alone is to be achieved. To estimate the values of G and A_w/A_0 required for propulsion, consider a cone-cylinder body with cone length-diameter ratio fixed at 4.0. The ratio $C_{D,1}/C_f$ for such a body, at zero angle of attack, is shown in figure 4 as a function of Mach number for several altitudes. (Base drag, boattail drag, and drag due to lift are not considered in this example.) The Reynolds number for these curves is based on a length of 50 feet. Friction coefficients for the laminar and turbulent cases were obtained from references 2 and 4, respectively.

Laminar curves are plotted to one-tenth the scale of the turbulent values. Figure 4 shows that the area ratio required by condition (10), for a given value of G, would be from 7 to 30 times as large (depending on altitude) for laminar flow as those required for turbulent flow.

Shown in figure 5 are the minimum values of G required by condition (10) for the case of fully turbulent boundary layer at altitudes of 35,000 and 100,000 feet. These values are plotted for ratios of wetted area to frontal area of 44 and 440. The former value corresponds to a cylindrical afterbody of length-diameter ratio of about 9; the latter value represents an extreme that might be attained by means of external fins. Superimposed on this plot are maximum values of G obtainable with hydrogen and helium for several values of the mean surface temperature T_w. In calculating these curves, it was assumed that the final gas temperature T_c was equal to T_w, and that $\varphi = \varphi_{max}$. The maximum specific impulses corresponding to the values of T_c are also indicated. Figure 5 shows that propulsion at a given Mach number can be accomplished with higher specific impulse the larger the area ratio and the higher the altitude. For large ratios of surface to frontal area, the effect of altitude becomes small.

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These conclusions are further illustrated in figure 6, wherein the specific impulses obtainable are cross-plotted against Mach number from figure 5. For the large area ratio $A_w/A_0 = 440$, hydrogen yields higher specific impulse than helium at all Mach numbers. For $A_w/A_0 = 44$, helium yields higher specific impulses than hydrogen for Mach numbers below 4.7 at 100,000 feet and below 6.2 at 35,000 feet. In the low supersonic Mach number range, and with $A_w/A_0 = 44$, aerodynamic heating alone cannot vaporize liquid hydrogen at a rate sufficient to provide a net thrust.

Figures 5 and 6 illustrate that the specific impulse of the coolant-vaporization method of propulsion increases as the ratio of surface to frontal area increases, and that the specific impulse increases with Mach number. At sufficiently high Mach numbers, the specific impulse obtainable with hydrogen vaporization can become comparable with values obtained with current liquid rocket propellants.

Heat-transfer requirements. - In the preceding section it was assumed that the mean surface temperature T_w was approximately equal to the final gas temperature T_c. Although T_w and T_c are arbitrary parameters in the thrust computation, their actual values depend on details of the internal heat transfer from the surface to the coolant.

From equations (1) and (2), a relation between T_w, T_c, and the weight flow w is obtained:

$$A_w \, \rho_0 \, V_0 \, c_{p,0} \, St_0(T_e - T_w) = w \left[h_v + c_{p,c}(T_c - T_v)\right] \tag{11}$$

It is necessary to determine whether a heat exchanger can be devised which will produce the desired or assigned values of T_w and T_c. Let A_i be the internal surface area exposed to the coolant (including 2888 fins and baffles) and let h_i and T_i be the mean heat-transfer coefficient from the internal surface to the coolant and the mean temperature of the coolant, respectively. Then, since the heat transferred into the external surfaces equals the heat transferred from the internal surfaces to the coolant,

$$A_w \, h_w(T_e - T_w) = A_i \, h_i(T_w - T_i) \tag{12}$$

In terms of Stanton number, the ratio of internal to external area becomes

$$\frac{A_i}{A_w} = \frac{\rho_0 \, V_0 \, c_{p,0} \, St_0(T_e - T_w)}{\rho_i \, V_i \, c_{p,i} \, St_i(T_w - T_i)} \tag{13}$$

But

$$\rho_i V_i Ap_i = w \tag{14}$$

where Ap_i is the internal flow passage area. Substitution of the value of w from equation (11) into equation (14) and the resulting value of $\rho_i V_i$ into equation (13) yields

$$\frac{A_i}{Ap_i} = \frac{h_v + c_{p,c}(T_c - T_v)}{c_{p,i} \, St_i(T_w - T_i)} \tag{15}$$

This equation shows that the relation between T_c and T_w is determined by the ratio of internal surface area to internal passage area, by the mean coolant temperature, and by the mean internal Stanton number. Inasmuch as a portion of the internal heat transfer takes place to a vaporizing liquid or solid, it is difficult to assign appropriate mean values of St_i and T_i. However, the order of magnitude of the ratio A_i/Ap_i required for specified T_w and T_c can be estimated by letting $T_i = \dfrac{T_c + T_v}{2}$. The value of St for heat transfer to the gas phase in pipes (ref. 5) is roughly 2 to 4×10^{-3}. If St_i is assumed to be 3×10^{-3}, equation (15) becomes

$$\frac{A_i}{Ap_i} = 333 \frac{h_v + c_{p,c}(T_c - T_v)}{c_{p,c}\left(T_w - \frac{T_c + T_v}{2}\right)} \tag{16}$$

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The minimum value of T_w that can be chosen is, as might be expected, $T_w \geq \frac{T_c + T_v}{2} \equiv T_i$. Attainment of this surface temperature requires infinite internal surface to passage area ratio.

The assumption $T_c = T_w$ yields

$$\frac{A_i}{Ap_i} = 667 \left[\frac{h_v}{c_p(T_c - T_v)} + 1\right] \tag{17}$$

For hydrogen, if $T_c \geq 200^\circ$ R, the maximum required value of A_i/Ap_i is about 1000. For $T_c = 50^\circ$ R, A_i/Ap_i is about 4000. These values are attainable with fins in the annular passage between inner and outer shell or by making the height of the annular passage sufficiently small relative to its length. It appears, therefore, that the values of T_c and T_w used in figures 5 and 6 should be attainable with suitable heat-exchanger design. Pressure drops required in such heat exchangers have not been considered. These are functions of the size of the missile, the flight Mach number, the altitude, and several other variables. At very high Mach numbers, where large coolant mass flows are required, the design of a heat exchanger with sufficiently low pressure drop may constitute a severe engineering problem.

Limitations on minimum ratio of nozzle-exit to chamber pressure. - Two possible limitations on the minimum attainable value of p_e/p_c exist. One is the requirement that the nozzle-exit pressure cannot fall below ambient static pressure without incurring thrust losses. The second is the condition that appreciable isentropic expansion below equilibrium condensation temperature is probably not attainable. Data obtained with air in hypersonic nozzles (ref. 6) indicate that extreme purity is required to attain even a few degrees of supersaturation. These data also indicate that the pressure-temperature curve for air tends to follow the equilibrium condensation curve when the isentropes intersect it. Since the expression given for φ (eq. (8)) assumes isentropic expansion, intersection of the isentrope with the condensation curve constitutes a limit beyond which I and G cannot be calculated by the equations derived herein.

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To determine the conditions under which the expansion may reach the condensation curve before ambient pressure is attained, the equilibrium condensation curve for hydrogen is plotted in figure 7, together with a sequence of isentropes. A flight altitude scale corresponding to ambient pressure is included on the right of this figure. The second isentrope to the right of the condensation curve forms a sort of boundary separating values of chamber pressure and temperatures for which the altitude limitation is more significant than the condensation limitation at flight altitudes less than about 100,000 feet. For initial chamber temperatures and pressures to the right of this isentrope, expansion through the nozzle can proceed isentropically to ambient pressure. To the left of this isentrope, the condensation curve may be encountered before ambient pressure is reached if the altitude is sufficiently high. If, for example, the initial chamber pressure is 1 atmosphere ($\log p = 0$), then for altitudes less than 100,000 feet and chamber temperatures greater than about 100° R, the ambient pressure is attained before the condensation curve is intersected.

To determine the penalty in thrust and specific impulse resulting from nonzero values of p_e/p_c and p_0/p_e, the ratio of the actual value of φ to the maximum value of φ is plotted for hydrogen in figure 8 for $p_0/p_e = 1$ and 0. These curves were obtained from equations (8) and (8a), with $\gamma_c = 1.5$. The ratio φ/φ_{max} is also the ratio of actual to ideal values of G and I, since both these parameters are directly proportional to φ.

It appears from figure 8 that values of φ/φ_{max} greater than 0.90 are attainable only with pressure ratios p_e/p_c less than about 0.007 if the nozzle-exit pressure is equal to ambient static pressure ($p_0/p_e = 1$). If the maximum chamber pressure is considered to be 10 atmospheres, ratios of p_e/p_c less than 0.007 are attainable only at altitudes greater than 60,000 feet. The numerical conclusions reached from figures 5 and 6 must therefore be modified according to figure 8 for lower altitudes or lower chamber pressures.

<u>Effect of specific gravity of propellant on range</u>. - Although hydrogen yields the best specific impulse of the coolants considered, the density of liquid hydrogen is very low. Hence, a given weight of hydrogen will occupy a much larger portion of a given missile volume than conventional liquid propellants. To obtain an indication of the relative effect of propellant density, consider the range equation for flight at constant velocity and constant lift-drag ratio (Breguet path):

$$X = V_0 \frac{L}{D} I \ln \left(1 + \frac{W_{0,p}}{W_e}\right) \tag{18}$$

where $W_{0,p}$ is initial propellant weight and W_e is initial total weight less $W_{0,p}$ (empty weight). Let ρ_p be the density of the propellant, v_p the initial volume occupied by the propellant, and v the total missile volume. Then equation (18) becomes

$$\frac{D}{L}\frac{X}{V_0} = I \ln\ (1+k\rho_p) \tag{19}$$

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where

$$k = \frac{v}{W_e}\frac{v_p}{v} = \frac{v_p}{W_e}$$

For given L/D, V_0, k, and I, the range is therefore proportional to $\log\ (1+k\rho_p)$.

If $\frac{D}{L}\frac{X}{V_0}$ is defined as a volume specific impulse I_{vol}, and if ρ_p is expressed in terms of specific gravity, equation (18) becomes

$$I_{vol} = \frac{D}{L}\frac{X}{V_0} = I \ln\ (1+k\rho_p) = I \ln \left[1 + 62.4\ k(\text{specific gravity})\right] \tag{19a}$$

This volume specific impulse, which determines the range at constant velocity, constant lift-drag ratio, and constant ratio of propellant volume to missile empty weight, is plotted in figure 9(a) for $k = 2.0$ cubic feet per pound and in figure 9(b) for $k = 0.10$ cubic feet per pound. Since range increases with k, large values of k correspond to long-range missiles, while small values of k suffice for short-range missiles. It is evident from figure 9 that the range for a given propellant volume is very sensitive to specific gravity, although specific gravity tends to become less important for long-range missiles. For $k = 2$ cubic feet per pound, liquid hydrogen (specific gravity ≈ 0.07) requires a specific impulse of 400 pound-seconds per pound to yield a range equal to that of an equal volume of propellant with specific gravity 0.7 and specific impulse of 200 pound-seconds per pound. These figures indicate that hydrogen vaporization, although it provides the best specific impulse, may be less satisfactory than other possible coolants if the structural weight and missile size required to hold the larger volume are considered. These conclusions, based on constant L/D, could be altered considerably if difference in fuel weight appreciably affects the optimum L/D or structural weight of the missile.

Design considerations for optimum range. - If the centrifugal force acting on a missile in steady horizontal flight is considered, the range equation (eq. (18)) becomes (if the rotation of the earth is neglected)

$$X = \frac{V_0 I \; L/D}{1 - \dfrac{V_0^2}{gR_0}} \ln\left(1 + \frac{W_{0,p}}{W_e}\right) \tag{20}$$

where R_0 is the radius of the earth. The lift-drag ratio of a wing-body combination can be written

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$$\frac{L}{D} = \frac{C_{L_d} A_d}{(C_{D,d} + 2C_{f,d})A_d + C_{D,b}A_0 + C_{f,b}A_{W,b}} \tag{21}$$

where the subscript d refers to wing values and subscript b refers to body values. It is assumed that interference between wing and body coefficients is negligible and that the body is at zero angle of attack regardless of the angle of attack of the wings. Each coefficient is based on the area by which it is multiplied, and $C_{D,d}$ and $C_{D,b}$ refer only to drag other than the friction drag.

For steady flight, the thrust equation (eq. (5)) becomes

$$\frac{A_W}{A_0}(G-1) = \frac{C_{D,1}}{C_f} \tag{22}$$

where $C_{D,1}$ includes, as before, all drag except the friction drag of the cooled surfaces. If only the body surface is cooled by the coolant, then the friction drag of the wings, as well as their pressure drag, must be included in $C_{D,1}$. If wings and body are cooled, $C_{D,1}$ is the pressure drag of the body and wings. Equation (22) therefore becomes

$$\overline{C}_f = \frac{C_{D,b} + \dfrac{A_d}{A_0} C_{D,2}}{G - 1} \tag{23}$$

where

$C_{D,2} = C_{D,d} + 2C_{f,d}$ if only the body is cooled

$C_{D,2} = C_{D,d}$ if body and wings are cooled

$\overline{C}_f = C_{f,b} \, A_{W,b}/A_0$ if only the body is cooled

$\overline{C}_f = C_{f,b} \, A_{W,b}/A_0 + 2C_{f,d} \, A_d/A_0$ if body and wings are cooled

Substitution of equation (23) into equation (21) yields

$$\frac{L}{D} = \frac{\left(1 - \frac{1}{G}\right) C_L}{C_{D,2} + \frac{A_0}{A_d} C_{D,b}} \tag{24}$$

For a given velocity, prescribed aerodynamic coefficients, and weight ratio, the range for a coolant-propelled missile is therefore proportional to $I(1-1/G)$ (eqs. (20) and (24)). From equations (6) and (7), this quantity can be written as follows: 2888

$$I - \frac{I}{G} = \frac{a_0}{g}\left[\varphi \sqrt{\frac{m_0}{\gamma_0 m_c} \frac{T_c}{T_0}} - \frac{M_0}{1.2} \frac{h_v + c_{p,c} T_0\left(\frac{T_c}{T_0} - \frac{T_v}{T_0}\right)}{c_{p,0} T_0\left(1+0.176 M_0^2 - \frac{T_w}{T_0}\right)}\right] \tag{25}$$

Plots of $(I-I/G)$ as functions of T_c for a given M_0 yield peak values of $(I-I/G)$ at an optimum value of T_c, to be denoted by $T_{c,opt}$. To obtain $T_{c,opt}$ as a function of Mach number, equation (25) was differentiated with respect to T_c. The resulting expression for $T_{c,opt}$ is

$$\frac{T_{c,opt}}{T_0} = \frac{\left(1+0.176 M_0^2 - \frac{T_w}{T_0}\right)^2}{b^2 M_0^2} \tag{26}$$

where

$$b^2 = \frac{4 \gamma_0 m_c}{1.44\, \varphi^2 m_0}\left(\frac{c_{p,c}}{c_{p,0}}\right)^2$$

For a given coolant, the optimum ratio of chamber temperature to ambient static temperature is then a function only of M_0 and of the mean surface temperature T_w. Since $(I-I/G)$ increases as T_w decreases, it is desirable to make T_w as small as possible. As pointed out in a previous section, a plausible minimum value for T_w is $T_w = T_c$. With this value, the expression for $T_{c,opt}$ becomes

$$\frac{T_{c,opt}}{T_0} = 1 + \left(0.176 + \frac{b^2}{2}\right) M_0^2 - \frac{1}{2} b M_0 \sqrt{4 + (0.704+b^2) M_0^2} \tag{27}$$

For hydrogen ($b^2 = 5.364$ for $\varphi = \varphi_{max}$), the variation of $T_{c,opt}$ with Mach number is shown in figure 10 for an ambient temperature of 392.4° R. It is noteworthy that the optimum chamber and surface temperatures are in the structurally acceptable range even at Mach numbers of the order of 20.

The maximum values of (I-I/G) corresponding to these optimum temperatures are also plotted in figure 10. With these values, the maximum values of I L/D can be obtained from equation (24) when the aerodynamic coefficients are specified. With I L/D determined, the range can be calculated as a function of velocity and fuel weight from equation (20).

In evaluating the range obtainable with coolant vaporization relative to other systems, it is necessary to consider that the wetted area required to yield a net thrust may be larger for the coolant-vaporization method than for conventional methods. Larger wetted areas may have a considerable effect in reducing the maximum lift-drag ratio, as can be seen from equation (21). The effect of this wetted area on L/D is included in equation (24). If the resulting L/D is less than for a conventional propulsion system, the specific impulse for the coolant-vaporization method must be correspondingly higher to yield the same range.

DISCUSSION

From the preceding sections, it appears that the vaporization of liquefied or solidified gases by aerodynamic heating can provide thrust greater than the friction drag of the cooled surfaces at all flight speeds if the coolant, the heat exchanger, and the pressure ratio are appropriately selected. This thrust is attained with no combustion, inlets, or machinery (other than, perhaps, a pump) and more than adequate surface cooling is provided at all flight speeds. The specific impulse associated with this thrust, however, is lower than that attainable with conventional rocket fuels unless liquid hydrogen is used as coolant and unless the flight Mach number is very high.

Detailed investigation of the conditions, or types of mission, for which coolant vaporization might be advantageously used as a sole means of propulsion is beyond the scope of the present report. The preliminary analysis presented herein does indicate, however, that flight in the atmosphere at extremely high Mach numbers (>10) can probably be accomplished efficiently with the coolant-vaporization system. Whether such flight paths are of interest remains to be established.

Although much of the analysis was concerned with use of coolant vaporization as an independent means of propulsion, utilization of this thrust source as an auxiliary propulsion system may have more immediate interest. As shown in figure 5, the specific impulse can remain fairly high for arbitrarily low Mach numbers if the value of G is reduced as 2888
the Mach number decreases. Suppose, for example, that the internal heat-transfer system is designed to make $T_W = T_c = 392.4^\circ$ R. Equation (17) indicates that such a heat-transfer system $\left(\text{fixed } A_i/Ap_i\right)$ can maintain this temperature at all Mach numbers if the mass flow w is adjusted according to equation (11). The specific impulse remains fixed at about 257 (fig. 1) at all speeds, and the parameter G varies as indicated by the straight line $T_c = 392.4^\circ$ R in figure 5. The value of G remains less than unity for Mach numbers less than about 7. For a given missile, therefore, the hydrogen-vaporization technique provides more and more thrust as the missile accelerates until, at a Mach number near 7, all friction drag is effectively cancelled. In addition, the surface remains cooled at no cost in propellant weight efficiency if the specific impulse of the main power plant is less than 257. For higher surface temperatures, the specific impulse obtained with liquid hydrogen can be even higher (see fig. 5).

An interesting distinction appears from equation (5) between design parameters desirable for $G > 1$ and those desirable for $G < 1$. If coolant vaporization is to be used primarily as an independent means of propulsion $(G > 1)$, large values of C_f are desirable so that a given net thrust can be attained with smaller values of A_w/A_0. If coolant vaporization is used primarily as an auxiliary power source $(G < 1)$, then small values of C_f are desirable. For this case, therefore, the possibility that the coolant may be able to stabilize a laminar boundary layer becomes significant. The curves presented in reference 7 indicate that complete stabilization of the laminar boundary layer may be feasible in the Mach number range 1.5 to 9 if the surface temperature can be reduced almost to the ambient static temperature. Although complete experimental verification of this result has not yet been obtained, experiments reported in references 8, 9, and 10 indicate that the transition Reynolds number is increased as the surface is progressively cooled. An evaluation of the relative efficiency of cooling with liquid hydrogen rather than with other systems should therefore include, in addition to the available thrust, the possibility that the friction drag may be reduced from the turbulent to the laminar value.

For aircraft designed for flight in the atmosphere with a conventional rocket power plant it appears that hydrogen vaporization can provide an extremely efficient cooling system since, as pointed out in the previous example, the specific impulse of the coolant flow can be of the same magnitude as that of the main power plant at all flight speeds. For use with air-breathing engines, however, it appears probable that other more efficient coolant systems can be devised. Since the specific impulse for air-breathing engines is much higher than for rockets, it would appear to be inefficient to carry large quantities of coolant from which specific impulses much lower than those of the main power plant are obtained.

Figure 8 shows that high ratios of chamber pressure to ambient pressure are required to attain values of specific impulse close to the maximum values. Since high chamber pressures involve heavy structures, the high ratios of chamber pressure to ambient pressure are best attained by flight at very high altitudes. Although the computations of the present report were limited, for convenience, to the isothermal region of the atmosphere, flight at greater altitudes by means of coolant vaporization may be feasible and desirable. The higher ambient temperatures in the altitude range from 150,000 to 200,000 feet make this range particularly desirable for coolant-vaporization propulsion. Equation (9) shows that, for a given T_c and T_W, the thrust increases as the square root of the ambient temperature. This equation assumes, however, that the skin friction and heat transfer are related according to continuum flow theory, and that the heat transfer from the surfaces due to radiation is negligible. Both of these assumptions must be reexamined for altitudes greater than 100,000 feet. The reduction in skin friction due to slip flow is beneficial if coolant vaporization is used as an auxiliary power source, but is harmful if coolant vaporization is to be used as an independent power source. Heat loss due to radiation reduces the thrust attainable because the mass flow for a given cooled surface area is reduced. Radiation loss can, of course, be kept small even at very high altitudes by using surfaces with very low emissivity.

It is apparent that further analysis is required to determine the best flight altitude for utilization of coolant vaporization to provide thrust.

CONCLUDING REMARKS

The preceding analysis indicates that vaporization of aircraft surface coolants by aerodynamic heating can, under certain conditions, produce a net thrust at all flight speeds. Liquid hydrogen, because of its low molecular weight, provides the highest specific impulse at the relatively low chamber temperatures characteristic of this propulsion method. Use of hydrogen vaporization as an independent propulsion system yields low specific impulses at low speeds; the specific impulse increases, however, as Mach number increases and can theoretically equal or exceed values obtainable with the best rocket propellants. The specific impulse of the coolant-vaporization propulsion system increases with altitude and with the ratio of cooled surface area to frontal area. A missile designed for efficient use of coolant-vaporization propulsion may therefore be one with thin external fins running longitudinally along the body. 2888

Use of hydrogen vaporization as an auxiliary propulsion system and as a means of maintaining constant surface temperature shows promise in connection with conventional rocket propulsion. Surface temperatures sufficiently low to preserve structural integrity can be maintained at all flight speeds, and the resulting gas can be ejected with a specific impulse comparable to that of conventional rocket propellants.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, May 6, 1953

APPENDIX A

SYMBOLS

A_0	maximum cross-sectional area of aircraft, ft^2
A_i	internal surface area in contact with coolant, ft^2
A_w	external surface area cooled by coolant, ft^2
Ap_i	internal passage area for coolant, ft^2
a_0	ambient speed of sound, ft/sec
b	$\dfrac{2\sqrt{\gamma_0\ m_c/m_0}}{1.2\ \varphi}\left(\dfrac{C_{p,c}}{C_{p,0}}\right)$
$C_{D,b}$	body pressure drag coefficient
$C_{D,d}$	wing pressure drag coefficient
$C_{D,l}$	total drag coefficient of aircraft less friction drag coefficient of cooled surfaces
C_f	mean external skin-friction coefficient
$C_{F,net}$	net thrust coefficient
C_L	lift coefficient
c_p	specific heat at constant pressure, Btu/lb-$^{\circ}$R
D	drag, lb
F_{net}	net axial force on missile
G	thrust parameter, defined by equation (6)
g	acceleration of gravity, 32.16 ft/sec^2
h	heat-transfer coefficient, Btu/sec-ft^2-$^{\circ}$R
h_i	internal mean heat-transfer coefficient, Btu/sec-ft^2-$^{\circ}$R

h_v heat of vaporization, Btu/lb

h_w external mean heat-transfer coefficient, Btu/sec-ft^2-$^{\circ}$R

I specific impulse, lb-sec/lb

I_{vol} volume specific impulse, sec 2888

k v_p/W_e

L lift, lb

M_0 flight Mach number

m_c molecular weight of coolant

m_0 molecular weight of air

P power

p static pressure, lb/ft^2

q_w mean heat-transfer rate from external boundary layer to external surface, Btu/sec-ft^2

R universal gas constant, $4.97 \times 10^4 \dfrac{\text{ft}^2}{\text{sec}^2} \dfrac{\text{lb}}{\text{lb-mole-}^{\circ}\text{R}}$

R_0 radius of earth, 20.9×10^6 ft

St Stanton number, $= h/\rho V c_p$

T static temperature, $^{\circ}$R

T_e $T_0 \left(1 + \dfrac{\gamma - 1}{2} \varepsilon M_0^2\right)$

T_v vaporization temperature of coolant at pressure p_c, $^{\circ}$R

t wing thickness ratio

V velocity, ft/sec

v	total missile volume, ft^3
v_p	initial propellant volume, ft^3
W_e	empty weight, lb
W_0	total initial missile weight, lb
$W_{0,p}$	total initial propellant weight, lb
w	coolant flow, lb/sec
X	range, ft or miles
α	angle of attack, radians
β	$\sqrt{M_0^2 - 1}$
γ	ratio of specific heats
ε	recovery factor
ρ	density, lb/ft^3
φ	function of γ_c, p_e/p_c, and p_0/p_e defined by equation (8)
φ_{max}	$= \sqrt{\frac{2\gamma_c}{\gamma_c - 1}}$

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Subscripts:

b	pertaining to body
c	values in collection chamber
d	pertaining to wing
e	nozzle exit
i	mean values for coolant
j	jet
opt	optimum
p	propellant
v	vaporization

w mean values at external surfaces

0 ambient stream conditions

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APPENDIX B

RELATION BETWEEN PROPULSIVE POWER AND RATE OF HEAT ADDITION

The power considerations for the coolant-propelled system are identical with those of the conventional rocket engine.

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$$P_j = \text{thrust} \times V_0 = IwV_0 = \frac{wV_jV_0}{g} \tag{B1}$$

The portion of the thermal energy of the propellant converted to kinetic energy in passing through the nozzle is

$$E_T = Jc_{p,c}(T_c - t_j) = \frac{V_j^2}{2g} \tag{B2}$$

where t_j is the propellant temperature following expansion through the nozzle. From equations (B1) and (B2), the ratio of propulsive power to thermal power (defined as weight flow times thermal energy used) is

$$\frac{P_j}{wE_T} = 2\,\frac{V_0}{V_j} \tag{B3}$$

Equation (B3) shows that, for rocket-type jets, the propulsive power can be less than, equal to, or greater than the thermal power of the propellant, depending on the ratio of missile velocity to effective jet velocity. This result is pertinent to the discussion in the text, since it can readily be shown that the rate of energy addition to the coolant due to aerodynamic heating is not sufficient to overcome the friction drag of the cooled surfaces at high speeds. The result obtained in equation (B3) indicates that, in spite of this inadequacy of the energy obtainable from aerodynamic heating, values of G greater than unity are compatible with the first law of thermodynamics.

To evaluate the source of the energy corresponding to the propulsive power, the kinetic energy acquired by the propellant during acceleration to velocity V_0 must be considered. The propulsive power does work relative to a stationary reference frame; consequently, the total energy change of the propellant relative to that reference frame is of significance. The total energy per pound of propellant in the chamber before ejection is

$$E_c = \frac{V_0^2}{2g} + Jc_{p,c}T_c \tag{B4}$$

After ejection, the total energy of the propellant is

$$E_j = \frac{(V_0 - V_j)^2}{2g} + Jc_{p,c}t_j \tag{B5}$$

The difference between these energies, multiplied by the weight flow, is, as might be expected, the propulsive power. Thus, 2888

$$w(E_c - E_j) = \frac{wV_0V_j}{g} \approx wIV_0 \tag{B6}$$

Equations (B5) and (B6) show that part of the propulsive power, relative to the stationary reference frame, arises from the kinetic energy acquired by the propellant, relative to that reference frame, during acceleration to velocity V_0.

At very low speeds, the kinetic energy of the propellant, as well as the aerodynamic heating and the friction drag, becomes negligible. The coolant-vaporization system then develops thrust only if the vaporization temperature is less than ambient air temperature. The atmosphere then serves as a heat source to vaporize the coolant, which develops pressure in the chamber and generates thrust in expanding through the nozzle.

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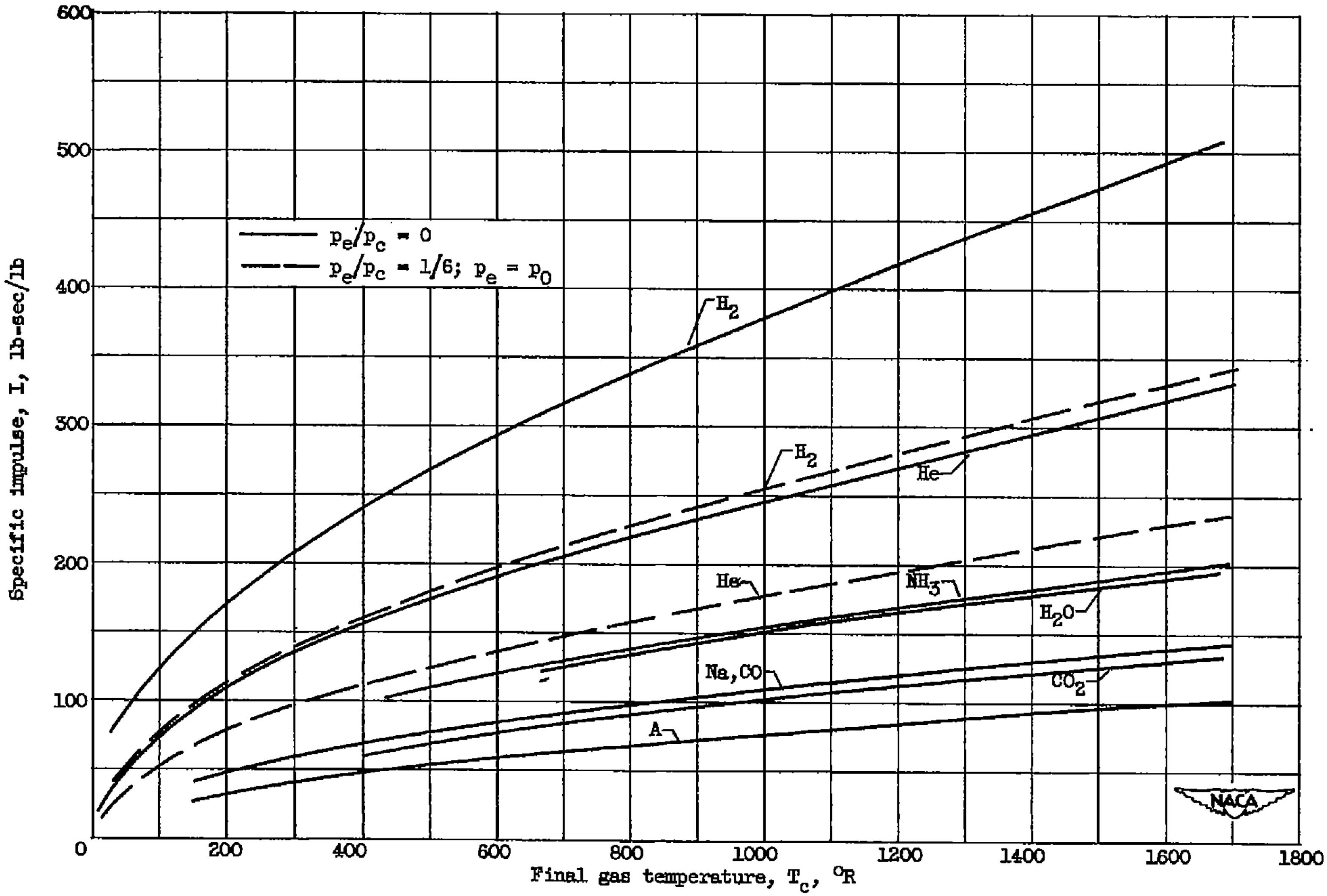


Figure 1. - Variation of specific impulse with final gas temperature.

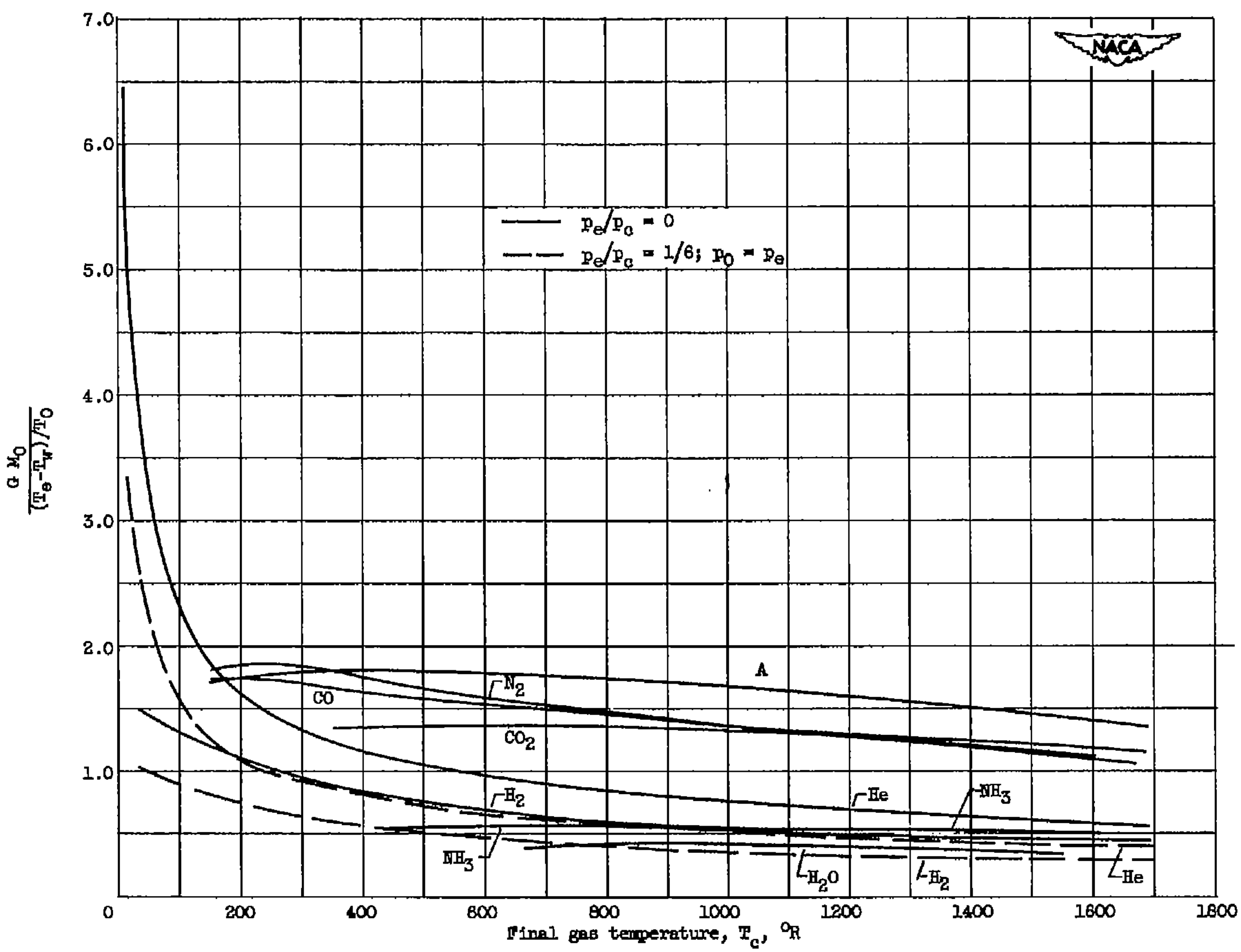
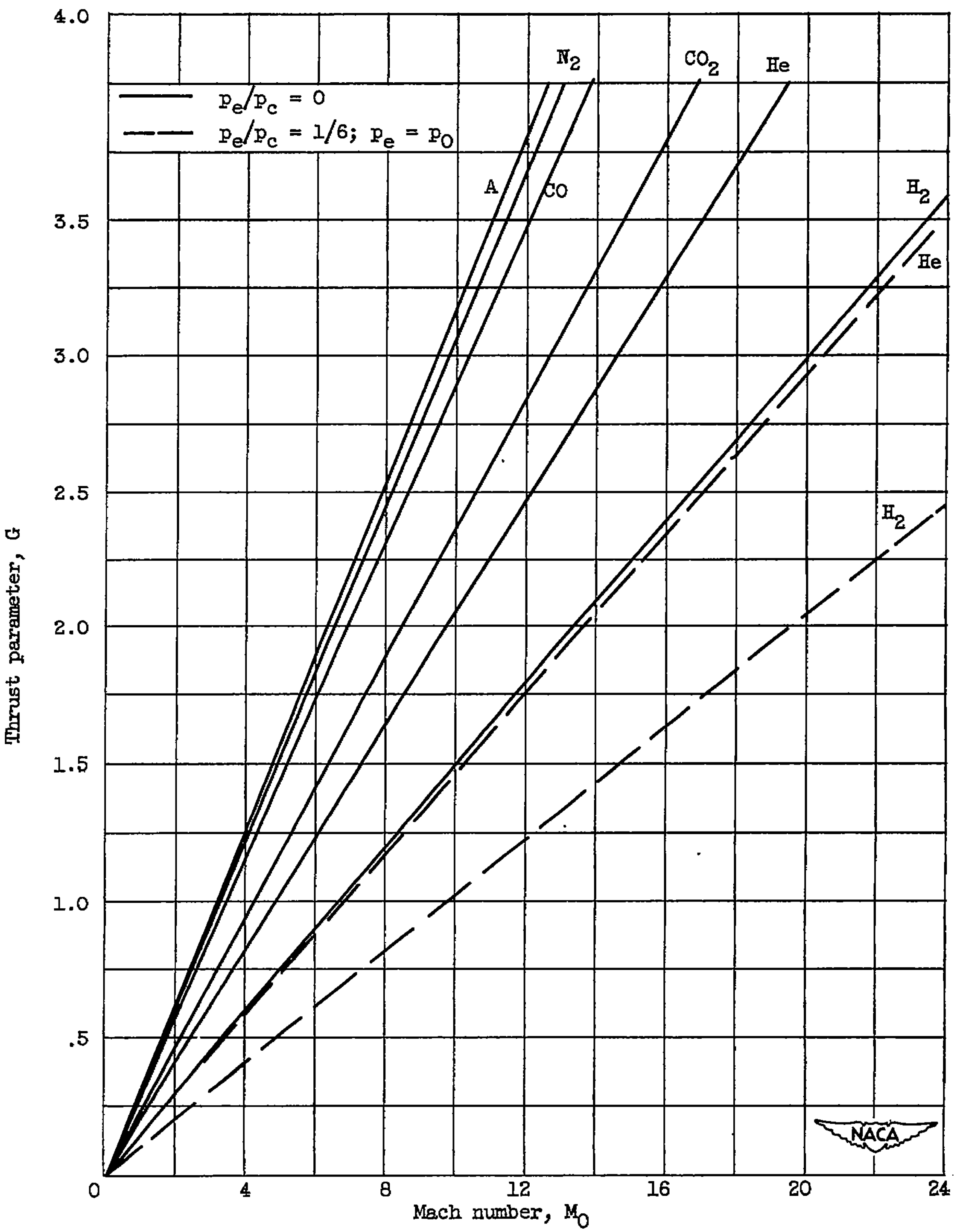


Figure 2. - Variation of thrust parameter with chamber temperature.



(a) $T_w = T_c = T_0$.

Figure 3. - Ratio of generated thrust to friction drag as function of Mach number for cooled external surfaces. $T_0 = 392.4^\circ$ R.

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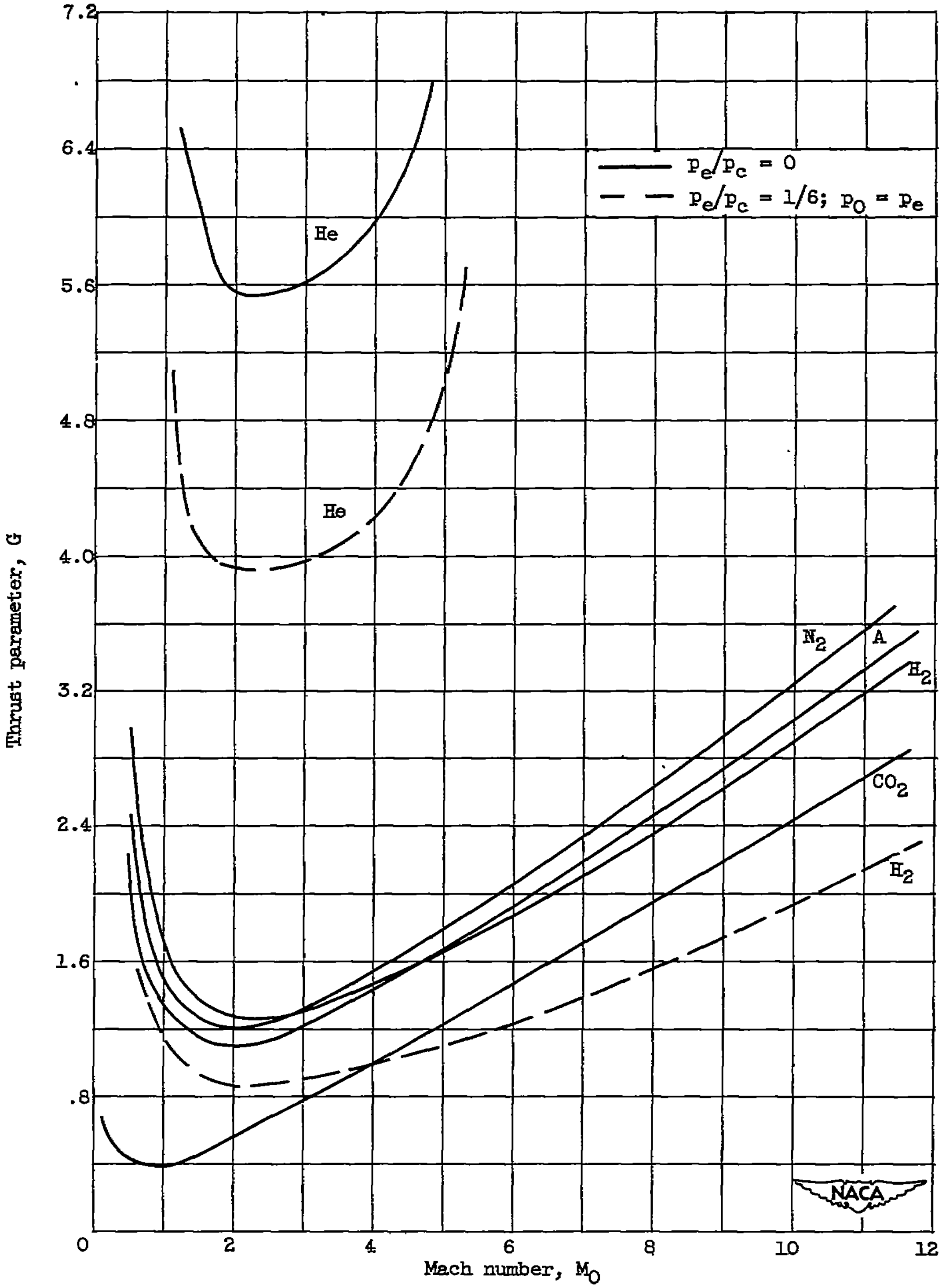
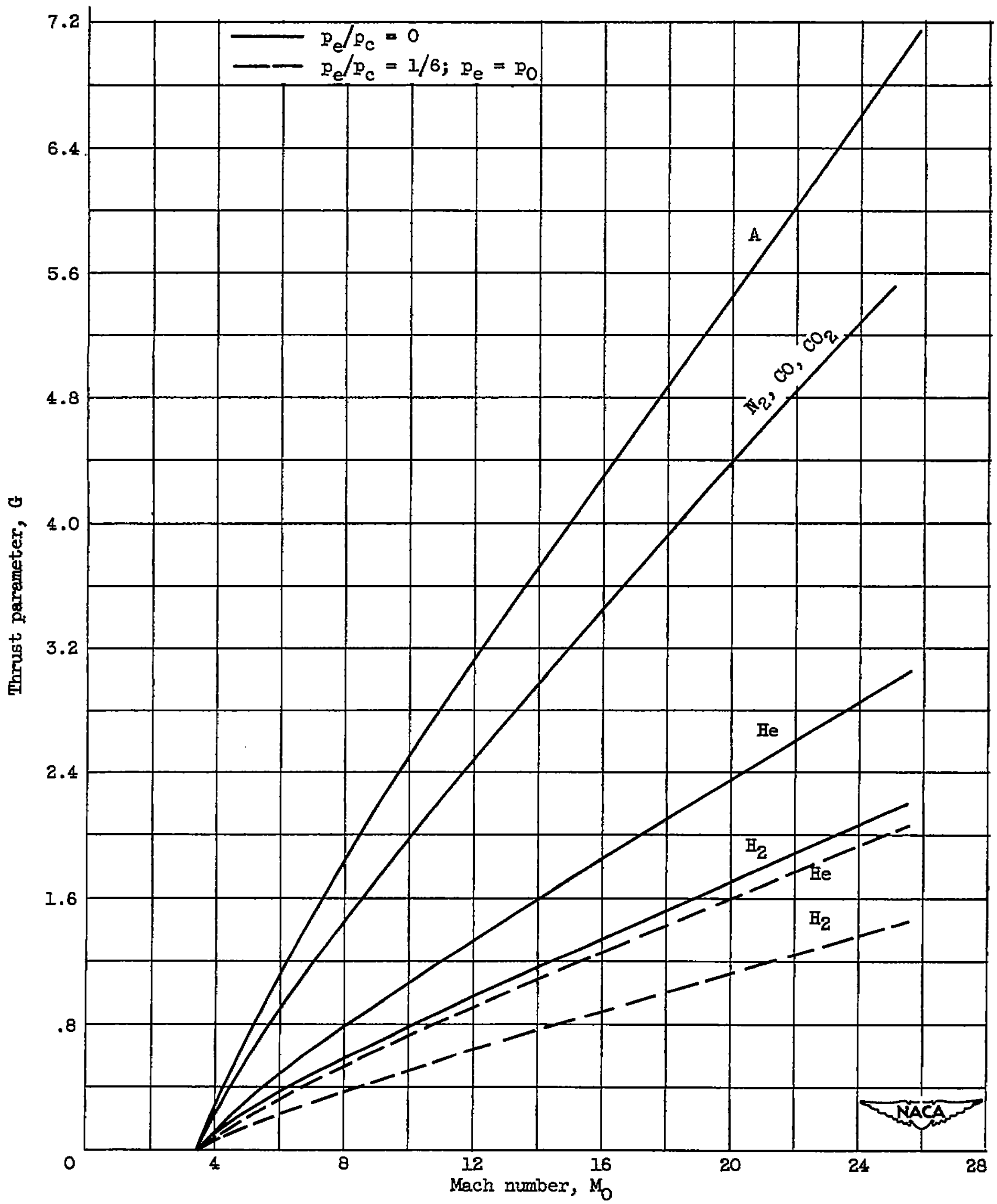


(b) $T_c = T_w = T_v$

Figure 3. - Continued. Ratio of generated thrust to friction drag as function of Mach number for cooled external surfaces. $T_0 = 392.4^\circ$ R.



(c) $T_c = T_w = 1200^\circ$ R.

Figure 3. - Concluded. Ratio of generated thrust to friction drag as function of Mach number for cooled external surfaces. $T_0 = 392.4^\circ$ R.

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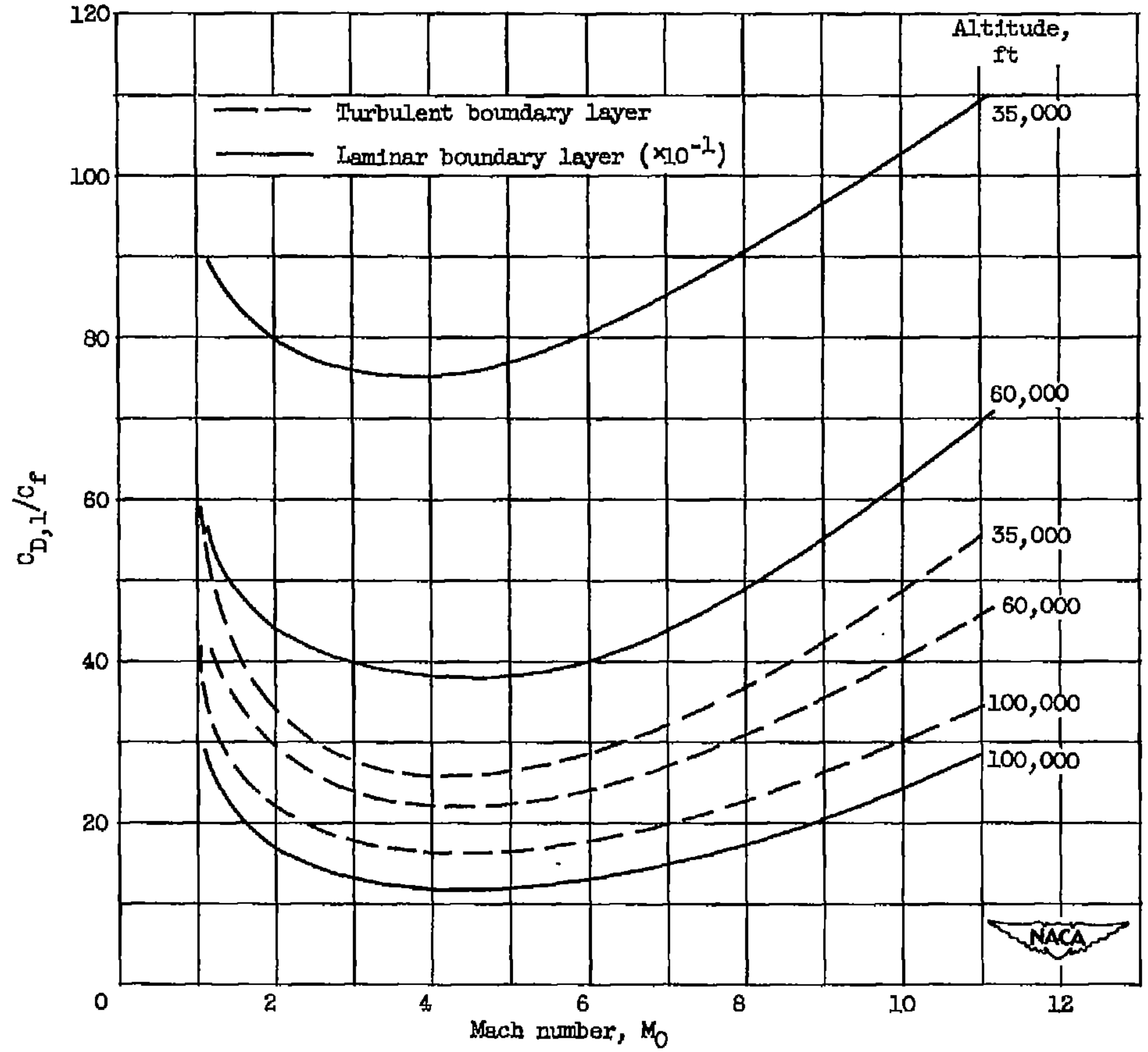

Figure 4. - Ratio of pressure-drag coefficient to mean skin friction coefficient for cone with length-diameter ratio = 4.0.

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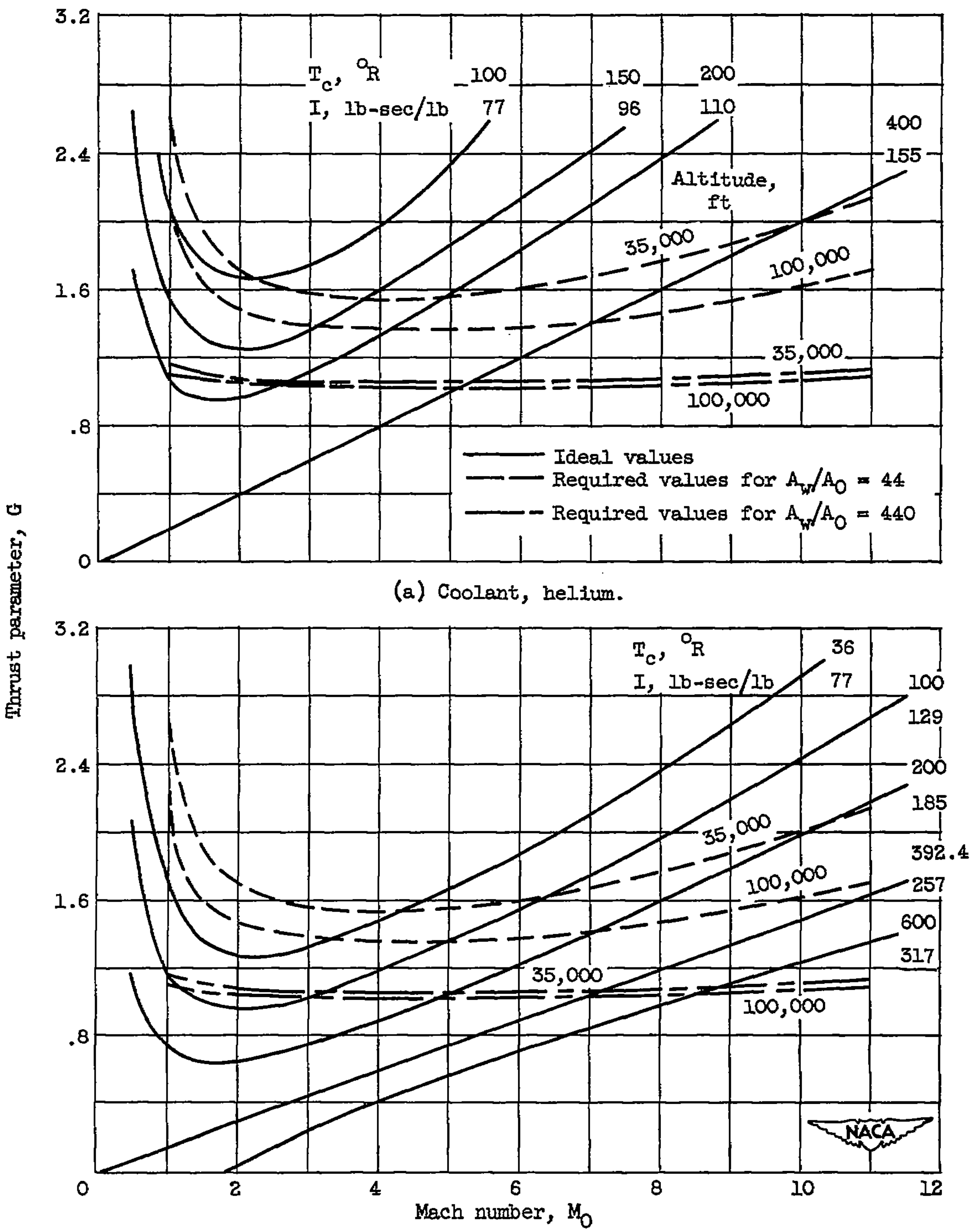


(b) Coolant, hydrogen.

Figure 5. - Required and available values of thrust parameter G for propulsion of cone-cylinder body. Turbulent boundary layer; $p_e/p_c = 0$; $T_w = T_c$.

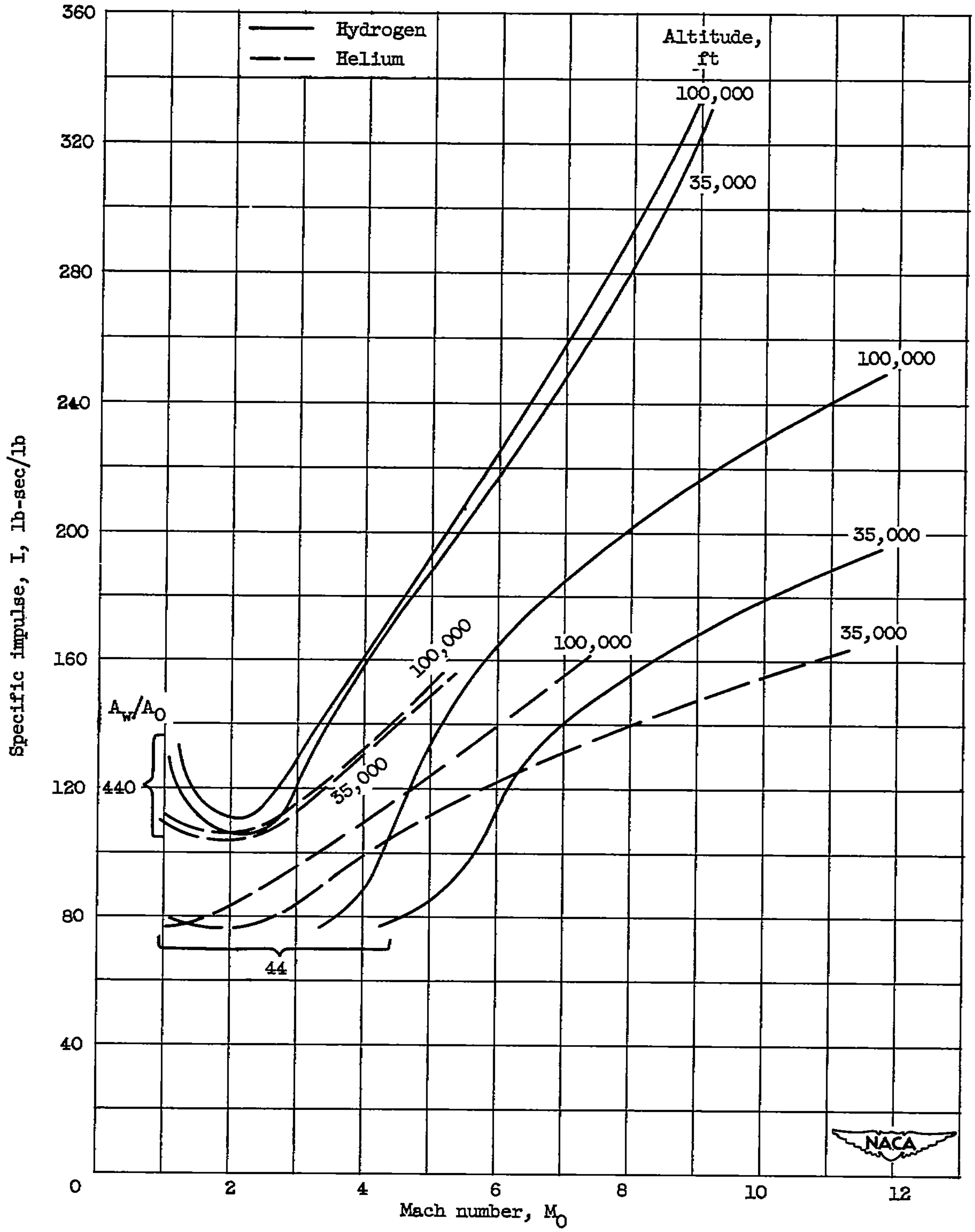


Figure 6. - Maximum specific impulse for propulsion of cone-cylinder body. $T_w = T_c$; $p_e/p_c \approx 0$; $T_0 = 392.4^\circ$ R.

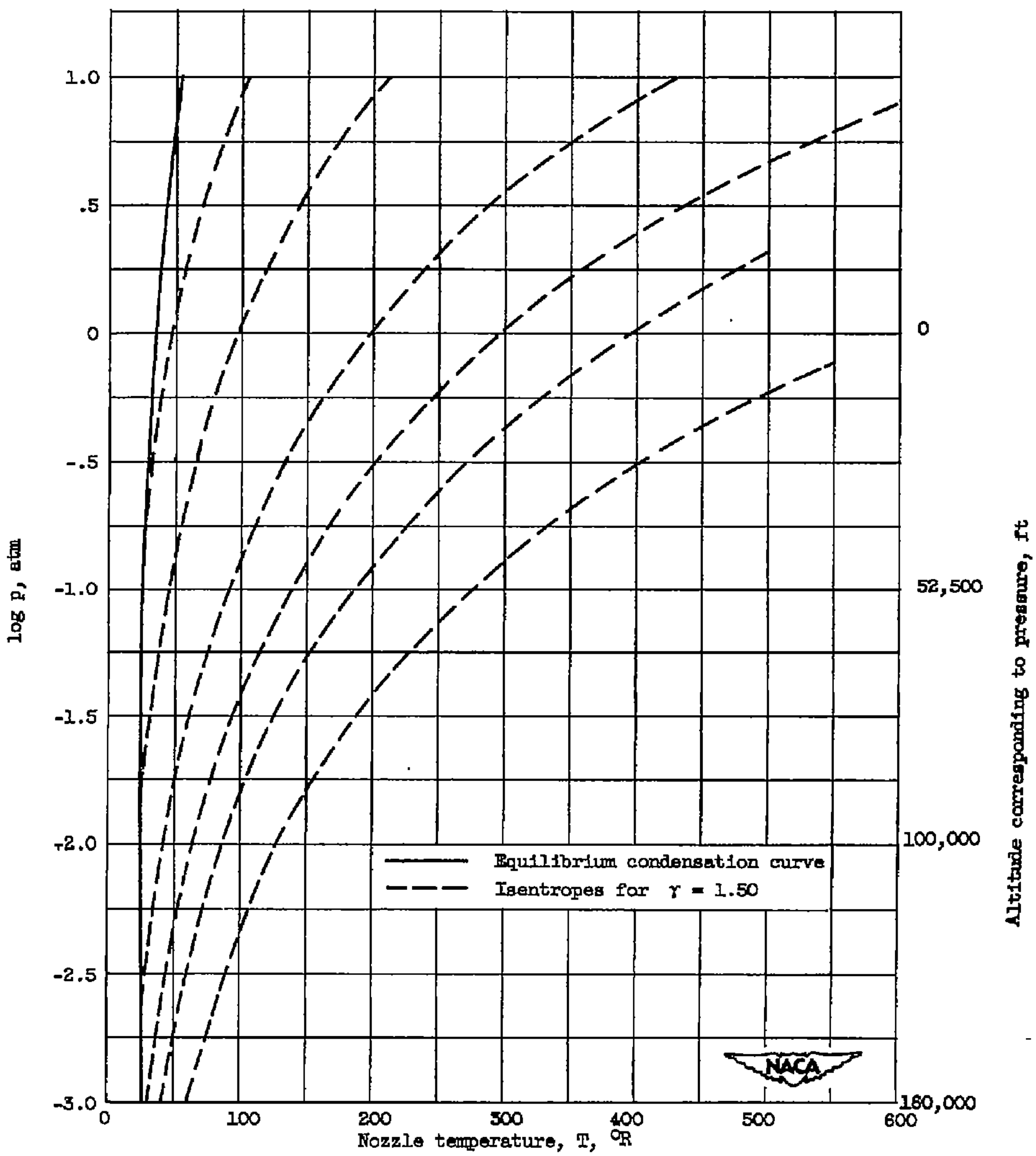


Figure 7. - Equilibrium condensation curve and isentropes for hydrogen.

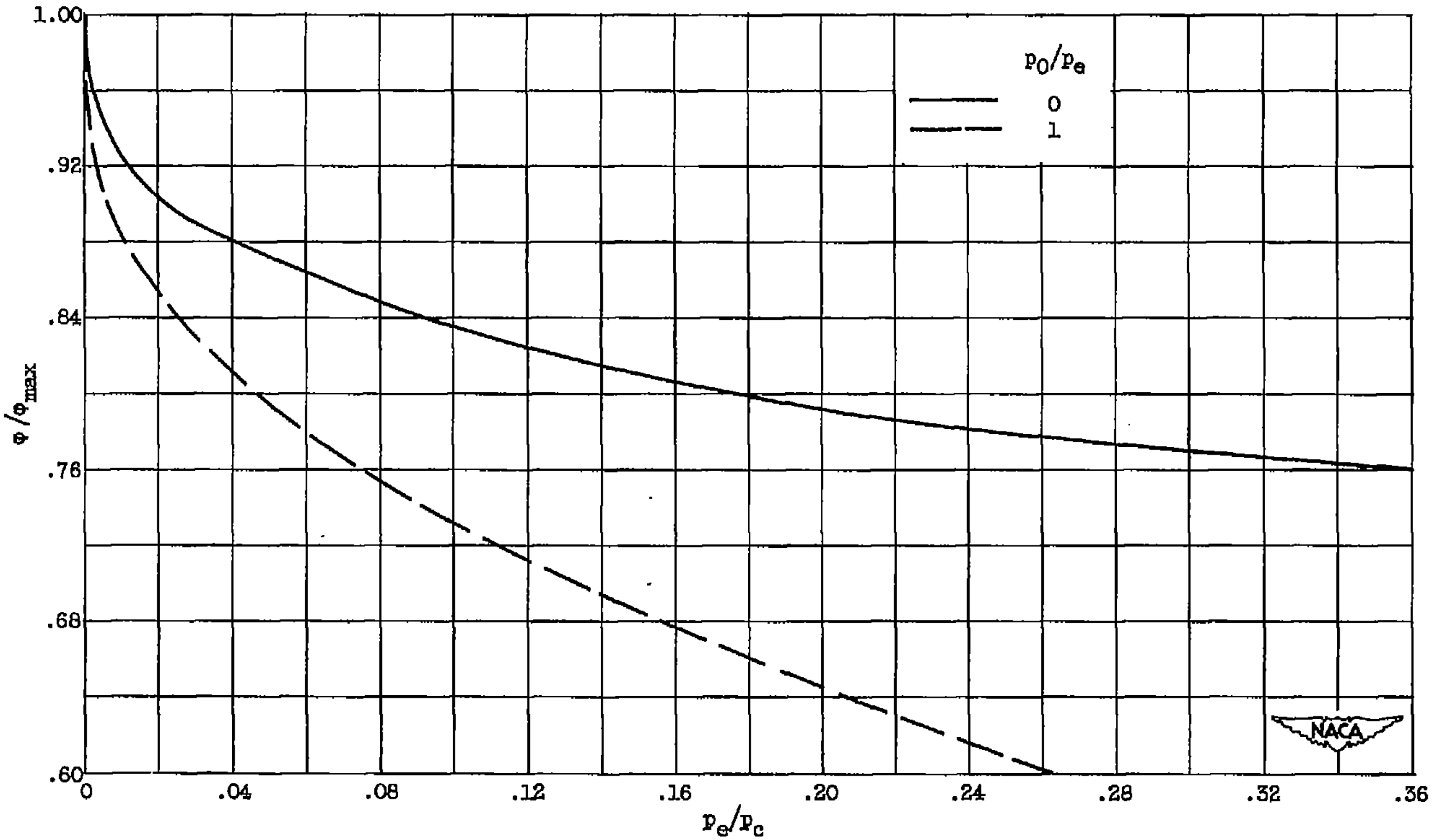
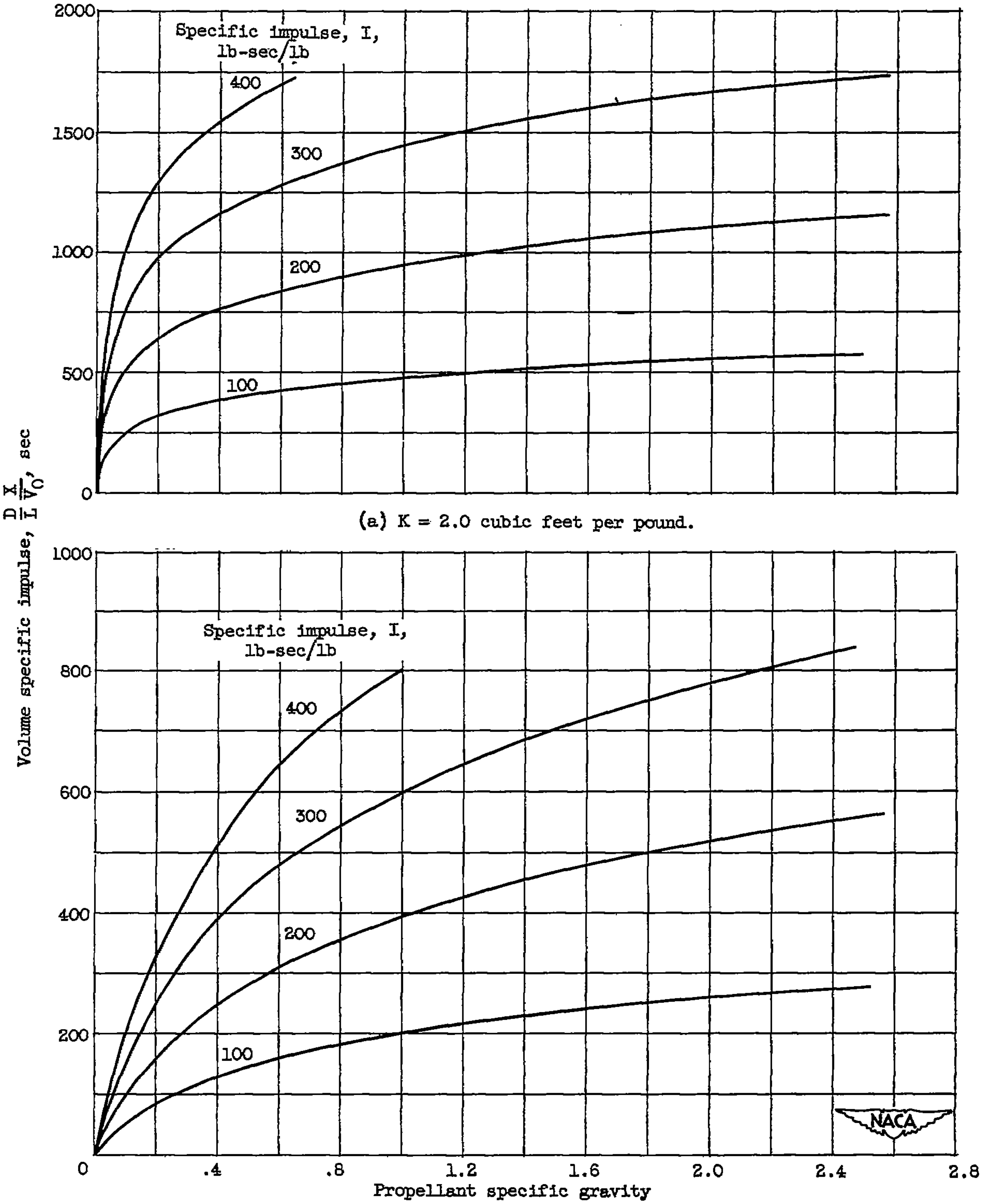
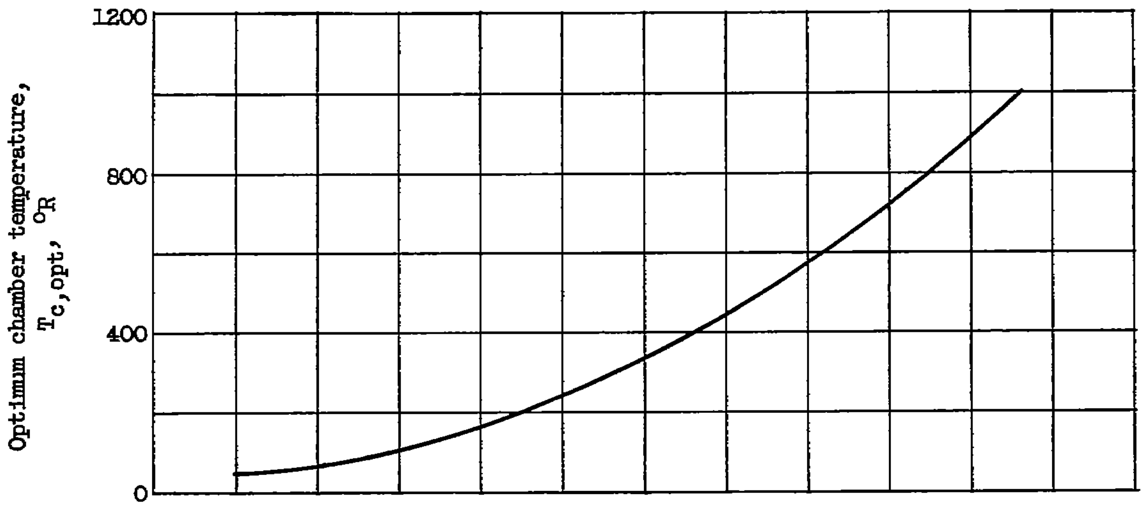


Figure 8. - Ratio of attainable to maximum value of φ as function of pressure ratio. $\gamma = 1.50$.

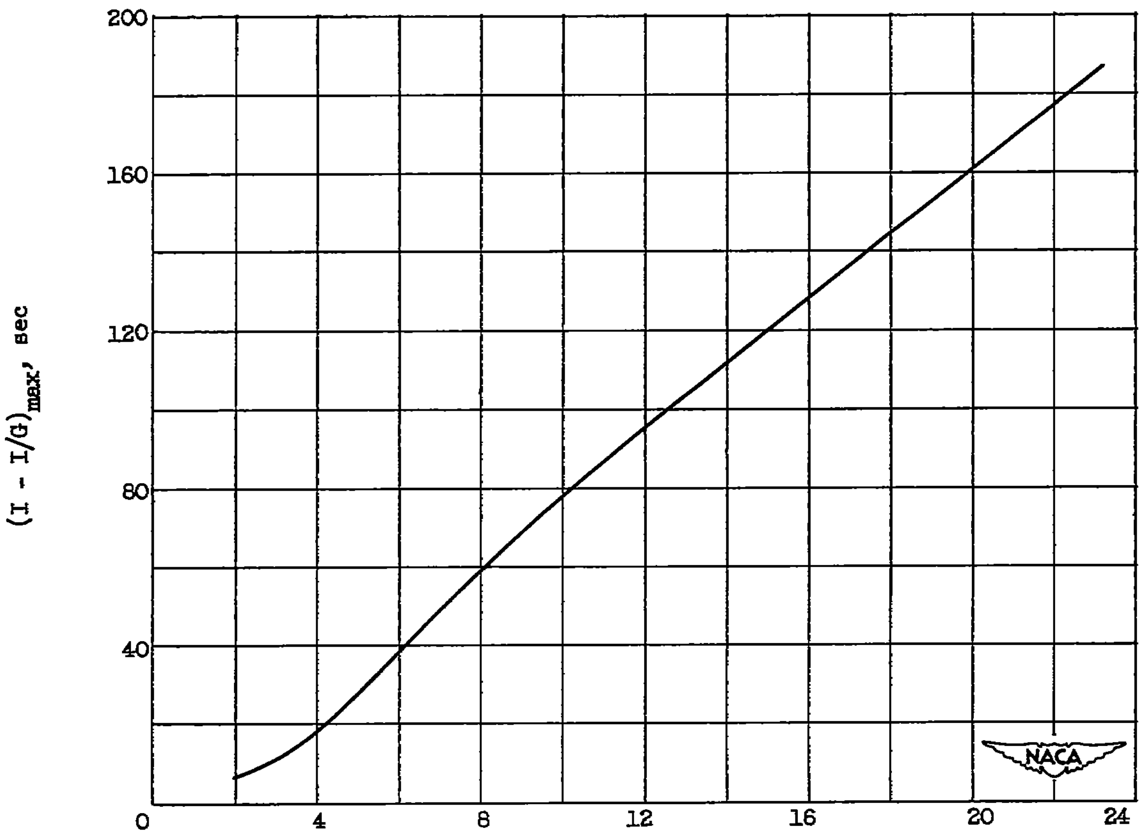


(b) K = 0.10 cubic feet per pound.

Figure 9. - Volume specific impulse as function of specific impulse and specific gravity.



(a) Optimum chamber (and surface) temperature.



(b) Maximum value of I - I/G.

Figure 10. - Maximum value of range parameter $(I - I/G)_{max}$ and corresponding values of chamber temperature for hydrogen cooling. $T_c = T_w$.

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